its construction. In the construction of the mathematical model of the subject note, the assumptions[4] were intended to be mutually consistent with those of linear elasticity theory. Regarding this, an order of magnitude analysis[5] shows that terms of the type $\Omega^2 u$ are of the same order as certain nonlinear terms, which are, of course, neglected in the linear theory. Hence, terms of the type $\Omega^2 u$, even though they are linear, were regarded as small and neglected. The centrifugal forces then produce a static deformation about which the vibratory motion takes place. Because the equations are linear, and only small deformation is considered, this static deformation may be a accounted for by superposing a particular solution of Eq. (3) of Ref. 4.

Third, a discussion that would consider the effects of terms of the type $\Omega^2 u$ and the effects of the initial static stress on the vibratory motion should, in order to be consistent, also include the notions of nonlinear strain, nonlinear stress-strain equations, initial and deformed coordinates, etc. It was not my intention to consider these in the subject note.

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Comments on "Bow Shock Shape about a Spherical Nose"

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In a recent Technical Note, Berman[1] has presented a simplified method to predict the bow shock-wave profile about a spherical nose for Mach numbers greater than five. The results of the analysis are contained in two expressions, one of which is more accurate and utilizes the factors of radius ratio, eccentricity, and shock detachment distance as functions of density ratio across a normal shock.

It is interesting to note that a similar correlation concept of the bow shock profile for spheres utilizing the density ratio across a normal shock was employed in Ref. 2 in a study of hypersonic blunt-body similitude. Gregorek and the present writer[2] began with the general form of the equation for the spherical shock as obtained from blast wave analogy, i.e.,

$$r_s/d_N = A(x/d_N)^n \tag{1}$$

where the coordinate system is identical to Ref. 1 with the exception that the origin is at the shock apex. It has been shown by previous results[3] that the usual values associated with A and n, arrived at by blast wave theory, are inadequate in the prediction of the shock-wave profiles about spheres for $x/d_N < 4$.

The approach used in Ref. 2 was to obtain photographs of the bow shock profiles about spheres by means of a glow discharge apparatus installed in the 4-in. continuous, freejet, hypersonic wind tunnel of The Ohio State University. These photographs were projected on a grid, and the nondimensionalized coordinates r_s/d_N and x/d_N were determined. When the data thus obtained are displayed on a logarithmic scale, the profiles may be observed to be nearly linear and may be expressed in the form of Eq. (1).

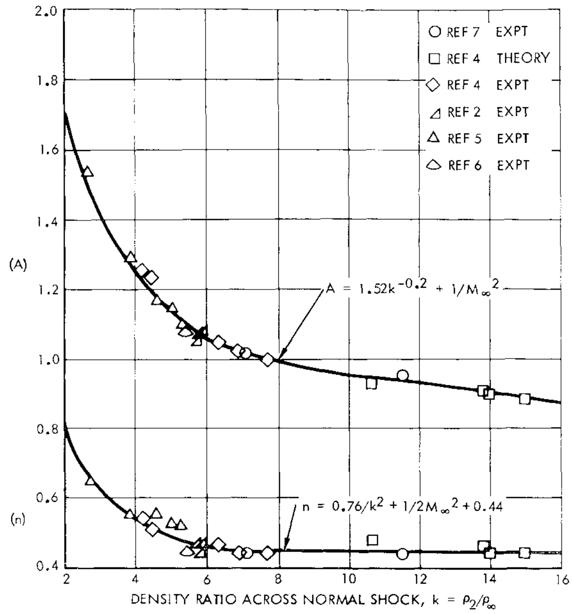

Fig. 1 Dependence of shock-wave constants on density ratio across normal shock.

Employing the shock-wave constants obtained for spheres from this study[2] and from sphere shock profiles presented in investigations by Seiff and Whiting,[4] Baer,[5] Love,[6] and Lobb,[7] values for A and n were plotted against density ratio as shown in Fig. 1. From these results, an empirical correlation for A and n based on density ratio and Mach number was formulated and took the form shown in Fig. 1. The experimental values of A are well represented by its empirical expression, with the maximum deviation of any of these points being less than 2%. Values of the exponent n show more

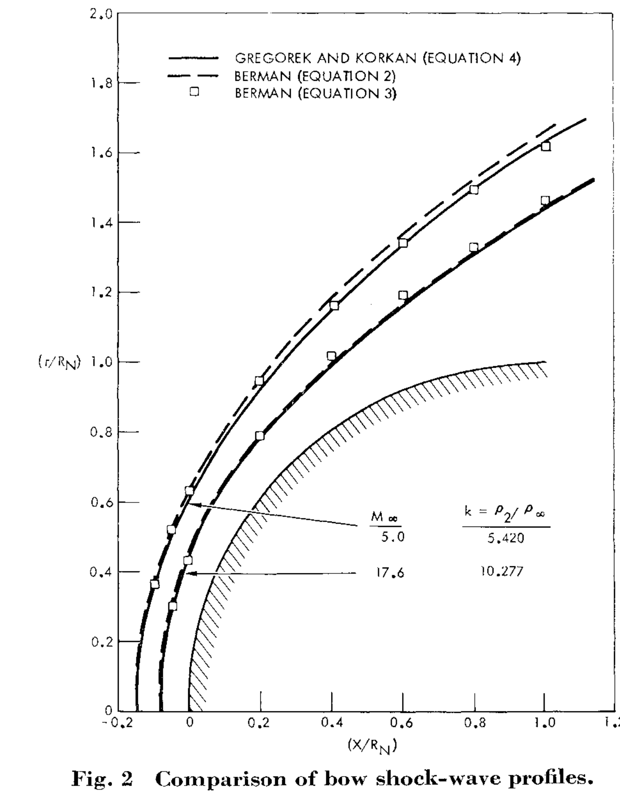

Fig. 2 Comparison of bow shock-wave profiles.

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scatter, with discrepancies of slightly greater than 10% being observed. This larger amount of scatter is due to the greater measurement difficulty in obtaining the slope of the shock from the experimental data.

Comparison of the results of these correlations and Ref. 1, using the specific flight conditions chosen by Berman, are shown in Fig. 2. Included for means of comparison is the accurate form of Berman's analysis, i.e.,

$$\left(\frac{r_S}{R_N}\right)^2 = 2\left(\frac{R_S}{R_N}\right)\left(\frac{X}{R_N} + \frac{\Delta}{R_N}\right) - e_s\left(\frac{X}{R_N} + \frac{\Delta}{R_N}\right)^2 \quad (2)$$

where

$$R_S/R_N = 2.09(1/k)^{0.1958} \qquad \Delta/R_N = 0.88(1/k)^{1.053}$$

with the values of the eccentricity factor e_s being expressed as functions of $1/k$ and specific locations of $(X/R_N + \Delta/R_N)$. In addition, Fig. 2 shows the results of Berman's approximate form, i.e., after approximating the eccentricity data with a single analytical expression and utilizing the forms for R_S/R_N and Δ/R_N as functions of density ratio, Eq. (2) was expressed as

$$\frac{r_s}{R_N} = \left\{4.18\left(\frac{1}{k}\right)^{0.1958}\left[\frac{X}{R_N} + 0.880\left(\frac{1}{k}\right)^{1.053}\right] - 0.646\left[\frac{X}{R_N} + 0.880\left(\frac{1}{k}\right)^{1.053}\right]^{1.467}\right\}^{0.5} \quad (3)$$

These expressions [Eqs. (2) and (3)] may be compared to the correlation obtained by Gregorek and the present writer, i.e.,

$$\frac{r_s}{d_N} = \left(1.52\, k^{-0.2} + \frac{1}{M_\infty{}^2}\right)\left(\frac{x}{d_N}\right)^{0.76k^{-2} + 1/(2M_\infty{}^2) + 0.44} \quad (4)$$

The results of these correlations, shown in Fig. 2, show relatively good agreement at Mach 5 and 17.6. However, the flight condition associated with the Mach 30 case indicated poor agreement because the limits of the empirical correlation of Ref. 2 had been exceeded, i.e., $k > 16$. Although the accuracy of Berman's results is unquestionable, the empirical correlation of Ref. 2 provides a comparatively simple expression extending over a wide range of density ratios, e.g., from 2 to 16, with an acceptable degree of accuracy. The advantage gained in using the results of Berman's approximate analysis [Eq. (3)] lies in the incorporation of the shock detachment distance in the over-all expression for the bow shock profile, whereas Eq. (4) does not contain this feature because of the choice of coordinate system. However, this easily may be rectified by use of Serbin's[8] or Ambrosio and Wortman's[9] results for values of the shock detachment distance.

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Comments on the Work Function of Metal Droplets

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The papers presented by Rowe and Kerrebrock[1] and by Smith[2] have been of interest to the author in connection with our investigation of the work function of small metallic particles. Using classical image theory, Rowe and Kerrebrock[3] and Smith[2] derived two slightly different but completely equivalent expressions for the work required to remove an electron from a metal droplet. Their respective results are (the original notations and definitions are employed here)

$$W_Z = \varphi_s + \tfrac{3}{8}(e^2/4\pi\epsilon_0 A) + (Z - 1)e^2/4\pi\epsilon_0 A \quad (1)$$

and

$$W = e\varphi_w + \tfrac{3}{8}(e^2/4r_d) + (Ze^2/r_d) \quad (2)$$

in which W_Z is defined as the work required to remove the Zth electron from an initially neutral metal droplet and W is defined as the work required to remove an electron from a metal droplet of Z times charged. The flat surface work functions are φ_s and $e\varphi_w$, A and r_d are the droplet radii, and the rest of the notation is conventional. The purpose of this comment is to call attention to relevant earlier work, which can be profitably combined with either Eq. (1) or Eq. (2) to form a more complete expression for calculating the energy required to remove an electron from a metal droplet.

According to the elementary theory of metals, the electron work function of a metal φ is $\varphi = W - E_F$, in which W is the image energy and E_F is the Fermi energy. Assuming this equation also applies to finely dispersed metallic particles and denoting the bulk and dispersed states by the subscripts 0 and 1, respectively, we obtain the following expression for the droplet work function:

$$\varphi_1 = \varphi_0 + (W_1 - W_0) - (E_{F1} - E_{F0}) \quad (3)$$

A comparison of Eqs. (1) and (2) with Eq. (3) shows that, whereas the authors of Refs. 2 and 3 have accounted for the contribution of $\varphi_0 + (W_1 - W_0)$ to W_Z and W, they have not considered the possible contribution of the term $(E_{F1} + E_{F0})$. We shall see that this term is quite important, especially for neutral droplets with diameters less than about 100 Å.

In Ref. 4 it was pointed out that if a metal is dispersed, the kinetic energy of the electrons would increase, and thus would raise the Fermi energy of the metal. Using the simple model of an electron gas in a three-dimensional potential box for a metal, Zhukhovitskii and Andreev derived a first-order expression for the effect of dispersion on the Fermi

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